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Research Article

Design Considerations on a Composite Cantilever Leaf Spring Made of Aluminum and Steel Materials

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Abstract

The purpose of this article is to explain the understanding of the finite element method step by step and to inspire the understanding of the main principle. To ensure easy understanding, the number of elements was limited, and a leaf spring design from different materials was examined. A composite is a structural material that consists of at least two combined constituents. To determine the strain and stress fields the finite element method is applied step by step and results are presented to assist in the design of such machine components. Plane triangular finite element model of the composite spring with three finite elements, all fundamental steps of stress analysis are considered in this article.

Keywords: Composite spring, finite element method, leaf spring design, stress fields, stress analysis.

Introduction

The purpose of this article is to explain the understanding of the Finite Element Method step by step and to inspire the understanding of the main principle. Various softwares are used for real problems in practice. In industrial application software such as ANSYS, NASTRAN and ABAQUS may be used. In this article, while the number of elements is only 3, for example, in design of turbine blade using with ANSIS 15 software, 4680 elements are used (Baqer *et al.*, 2019). Here the number of elements is kept small to facilitate the comprehension of the method. The main purpose of springs is to absorb, store and release energy. Leaf plays a vital role in supporting lateral loads, shock loads, brake torque, and driving torque. When flexibility or deflection in a mechanical system is specifically desired, some form of spring can be used. Otherwise the elastic deformation of an engineering body is usually a disadvantage (Spotts and Shoup, 2006).

Advantages of leaf spring over helical spring are that the ends of the springs are guided along a definite path so as to act as a structural member in addition to shock absorbing device. This is the reason why leaf springs are still used widely in a variety of automobiles. Leaf spring made of composite materials made it possible to reduce the weight without any increase in load carrying capacity and stiffness of the leaf spring. Therefore, analysis of composite material leaf springs has become essential in showing the comparative results with conventional leaf springs (Manchanda *et al.*, 2015; Kumar *et al.*, 2016).

A composite is a structural material that consists of at least two combined constituents that are combined at macroscopic level and not soluble in each other. Machine design is the art of planning or devising new or improved machines to accomplish specific purposes. In general, a machine will consist of several different mechanical elements properly designed and arranged to work together, as a whole. The principles of design are of course universal. The same theory or equations may be applied to a very small part, as an instrument or a larger but similar part used in a piece of heavy equipment. The stresses and deformations of rectangular leaf springs for small deflections can be found by the appropriate equations. A leaf spring can be considered as a cantilever beam under the bending stress. It has been shown that for most engineering purposes a beam which has a width less than ten times its thickness can be considered as narrow and thus bending theory can be applied (Foster, 1982; Timoshenko and Goodier, 2005). A theory of the bending of a beam has been presented that the solution agrees with the St. Venant solution when strains are small enough i.e. around one thousandth (Shield, 1984). Also three dimensional bending problems are investigated (Oja *et al.*, 1982). Important uses of beams made of several different materials occur in practice. Wooden beams are often reinforced by metal straps, and concrete beams are reinforced with steel rods. Weibull theory for wooden members has been expanded (Lui, 1981).

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Materials and methods

Experimental design: There are three main domains of learning; these are cognitive (thinking), affective (emotion/feeling), and psychomotor (physical/kinesthetic). By means of very user-friendly software the individual's ability develops to use sensory cues to guide motor activity ranging from sensory stimulation, through cue selection, to translation. In other words, the individual's perception (awareness) proceeds without an improvement in the cognitive (thinking) field.Individuals using such software are exposed to the guided response which is the early stages in learning a complex skill that includes imitation and trial and error and adequacy of performance is achieved by practicing. The individual using very user-friendly software achieves complex overt response (expert) which is the skillful performance of motor acts that involve complex movement patterns. Gained proficiency is indicated by a quick, accurate, and highly coordinated performance, requiring a minimum of energy. The thinking (cognitive) domain involves knowledge and the development of intellectual skills which includes the recall or recognition of specific facts, procedural patterns, and concepts that serve in the development of intellectual abilities and skills. There are six major categories of cognitive an processes, starting from the simplest to the most complex: "remember, understand, apply, analyze, evaluate and create". The individual using very user-friendly software specializes in reaching complex overt response in the psychomotor domain. However since the most complex analysis, synthesis and decision making performed by the smart and user friendly software in background (as embedded system approach) the individual begins to forget and lose thinking (cognitive) subsets, starting from top to down. In this study, in order to ensure the presence of the individual in the cognitive domain, not to lose his ability to think, the construction of a problem, preparation for solution with finite elements, preparation of mesh or grid generation, modeling and selection of the solution method are explained from the background.

Results and discussion

Application of the finite element method to the composite spring: The accuracy of the finite element method depends on both the type of the problem and upon the number and type of elements selected. To show easy understand the fundamentals principles of the method, let us use limited number of elements such as three, and plane triangle type of elements. Consider a composite spring shown in Fig. 1. Using the finite element method determine the strains and stresses for each material of that machine element, supported and loaded as shown in the figure. Aluminum and steel material properties are given in Table 1.

Fig. 1. Dimensions and loading of the composite spring.

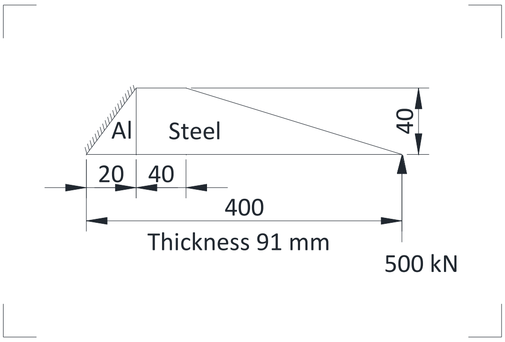


Table 1. Composite spring material properties.

Material	Yield strength, MPa	Modulus of elasticity, GPa	Poisson's ratio
Aluminum Al 7475	468	70	0.3
Steel AISI 9262	950	210	0.3

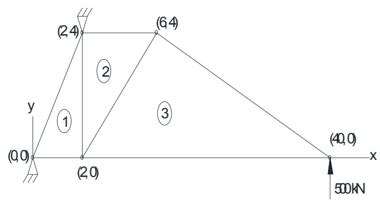
In general the sequence in the finite element application consists of 6 steps as follow (Seshu, 2012; Chandrupatla and Belegundu, 2015; Rao, 2017):

1. Sketch the structure and divide into elements
2. Indicate nodal displacements and equivalent nodal forces
3. Form the strain shape function, matrix B
4. Form the elasticity matrix, matrix D
5. Form the element stiffness matrix, matrix K^e
6. Combine the element stiffness matrices and compute strains and stresses

Finite elements

The leaf composite spring divided into three elements as shown in Fig. 2.

Fig. 2. Finite element representation of composite spring.



Nodal displacements

Each node has 2 degree of freedom. There are 5 nodes. Nodal displacements and applied external force of the composite spring are shown in Fig. 3.

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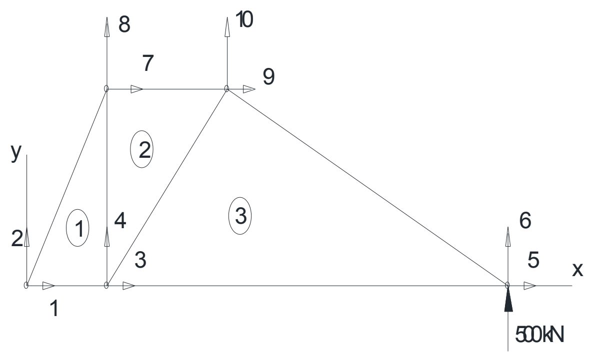
Fig. 3. Nodal displacements and force.



Finite elements shape functions

Strain shape function, B is:

$$B=Q\,C^{-1} \quad (1)$$

The matrix q consists of derivatives of the shape function, P

$$q=\begin{bmatrix} \frac{\partial P_1}{\partial x} \\ \frac{\partial P_2}{\partial y} \\ \frac{\partial P_1}{\partial y}+\frac{\partial P_2}{\partial x} \end{bmatrix} \quad (2)$$

Where P_1=[1 x y 0 0 0] and P_2=[0 0 0 1 x y]

Then the matrix Q is obtained as follow:

$$Q=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \quad (3)$$

Shape function for an element is given by C:

$$C=\begin{bmatrix} 1 & x_i & y_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_i & y_i \\ 1 & x_j & y_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_j & y_j \\ 1 & x_m & y_m & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & x_m & y_m \end{bmatrix} \quad (4)$$

For element 1, shape function matrix C is obtained as follow:

$$C_1=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 \\ 1 & 2 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 4 \end{bmatrix} \quad (5)$$

Thus strain shape function matrices B_1, B_2 and B_3 are obtained as follow:

$$B_1=\begin{bmatrix} -0.5 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.25 & 0 & -0.25 \\ 0 & -0.5 & -0.25 & 0.5 & 0.25 & 0 \end{bmatrix}$$

$$B_2=\begin{bmatrix} 0 & 0 & 0.25 & 0 & -0.25 & 0 \\ 0 & -0.25 & 0 & -0.25 & 0 & 0.25 \\ -0.25 & 0 & 0 & 0.25 & 0.25 & -0.25 \end{bmatrix} \quad (6)$$

$$B_3=\begin{bmatrix} -0.026 & 0 & 0.026 & 0 & 0 & 0 \\ 0 & -0.224 & 0 & -0.026 & 0 & 0.25 \\ -0.224 & -0.026 & -0.026 & 0.026 & 0.25 & 0 \end{bmatrix}$$

Elasticity matrix for finite element model

D for the plane stress case the elasticity matrix is given by

$$D=\frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \quad (7)$$

Where E and ν represent elastic modulus and Poisson's ratio for the spring material respectively.

Stiffness matrix for finite element model

Form the element stiffness matrix, K^e:

$$K^e=\int_{v^e} B^T D\, B\, d(vol) = B^T D\, B\, t\, \Delta \quad (8)$$

Where t and Δ are the thickness and the area of the element and B^T is the transpose of matrix B .

$$\begin{matrix} K_{ii}=B_i^T D\, B_i\, t\,\Delta & K_{ji}=B_j^T D\, B_i\, t\,\Delta & K_{mi}=B_m^T D\, B_i\, t\,\Delta \\ K_{ij}=B_i^T D\, B_j\, t\,\Delta & K_{jj}=B_j^T D\, B_j\, t\,\Delta & K_{mj}=B_m^T D\, B_j\, t\,\Delta \\ K_{im}=B_i^T D\, B_m\, t\,\Delta & K_{jm}=B_j^T D\, B_m\, t\,\Delta & K_{mm}=B_m^T D\, B_m\, t\,\Delta \end{matrix} \quad (9)$$

In the expressions above B_i represent the first two columns of matrix B. Bj the third and fourth columns of matrix B, and B_m the two last columns. Thus,

$$B=[B_i\ B_j\ B_m] \quad (10)$$

For element 1 following equation may be written:

$$K_{11}=\begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 0 & -0.5 \end{bmatrix}\frac{70*10^3}{1-0.3^2}\begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & \frac{1-0.3}{2} \end{bmatrix}\begin{bmatrix} -0.5 & 0 \\ 0 & 0 \\ 0 & -0.5 \end{bmatrix} 91*20*40/2 \quad (11)$$

It is obvious that for the steel elements 2 and 3 material modulus of elasticity should be used as 210 GPa. Thus all the other sub-matrices are now calculated in similar manner, and the stiffness matrix K^1 for element 1 is:

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$$K1=\begin{bmatrix} K_{ii} & K_{ij} & K_{im} \\ K_{ji} & K_{jj} & K_{jm} \\ K_{mi} & K_{mj} & K_{mm} \end{bmatrix} \qquad (12)$$

The stiffness matrices for element 2 and 3 can be obtained in similar manner.

Combining the element stiffness matrices of the composite spring model

Combine the element stiffness matrices and compute strains and stresses. The combined element stiffness matrix can be evaluated as follow:

$$K=2.8{*}10^{8}\begin{bmatrix} 35.44 & -4.98 & 0.08 & -0.1 & -1.11 & -17.11 & -16 & 2.8 & -18.4 & 19.4 \\ -4.98 & 61.04 & -0.09 & 0.32 & -14.7 & -3.2 & 2.4 & -5.6 & 17.37 & -52.56 \\ 0.08 & -0.09 & 0.05 & -0.02 & -0.13 & 0.11 & & & & \\ -0.1 & 0.32 & -0.02 & 0.05 & 0.13 & -0.37 & & & & \\ -1.12 & -14.12 & -0.13 & 0.13 & 49.81 & & & & -48.56 & 14.57 \\ -7.11 & -3.2 & 0.11 & -0.37 & & 20.57 & & & 17 & -17 \\ -16 & 2.4 & & & & & 16 & & & -2.4 \\ 2.8 & -5.6 & & & & & & 5.6 & -2.8 & \\ -18.4 & 17.37 & & & -48.56 & 17 & & 2.8 & 66.96 & -31.56 \\ 19.4 & -52.56 & & & 14.57 & -17 & -2.4 & & -31.6 & 69.56 \end{bmatrix} \qquad (13)$$

Knowing:

$$K=\begin{bmatrix} K_{dd} & K_{ds} \\ K_{sd} & K_{ss} \end{bmatrix} \qquad (14)$$

K_{dd}^{-1} is obtained as follow,

$$K_{dd}^{-1}=\frac{1}{2.8*10^{8}}\begin{bmatrix} 5.92 & 0.563 & -0.47 & 51 & 0.162 & 5.95 \\ 0.563 & 1.902 & -0.59 & -8.25 & 0.594 & 0.619 \\ -0.468 & -0.594 & 2.392 & 983 & 3.49 & 4.41 \\ 51.63 & -8.25 & 983 & 3273 & -7.26 & 95.28 \\ 0.163 & 0.594 & 3.49 & -7.25 & 2.21 & 0.079 \\ 5.95 & 0.619 & 4.41 & 95.3 & 0.08 & 11.59 \end{bmatrix} \qquad (15)$$

The vector of applied nodal forces f_c is,

$$f_c=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{matrix} * 10^{5}\ [N] \qquad (16)$$

Rearranging the above matrix yields;

$$K_{dd}^{-1}f_c=\begin{bmatrix} f_{cd} \\ f_{cs} \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \\ 0 \\ 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \\ 9 \\ 10 \\ 1 \\ 2 \\ 7 \\ 8 \end{matrix} * 10^{5}\ [N] \text{ and } a=\begin{bmatrix} a_d \\ a_s \end{bmatrix}= =\begin{bmatrix} a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_9 \\ a_{10} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \\ 9 \\ 10 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} * 10^{5}\ [N] \qquad (17)$$

Where letter a represent the deformations. A_d can be expressed in the following form:

$$a_d=K_{dd}^{-1}\ [f_{cd}][-K_{ds}][f_{cs}]=\begin{bmatrix} 9.22 \\ -1.47 \\ 176 \\ 585 \\ -1.29 \\ 17.0 \end{bmatrix}\begin{matrix} 3 \\ 4 \\ 5 \\ 6 \\ 9 \\ 10 \end{matrix} {*}10^{-2}\ [mm] \qquad (18)$$

The column vector f_{sr} is given by

$$f_{sr}=-f_{cs}+K_{ds}\,a_d+K_{ss}\,a_s=\begin{bmatrix} -151 \\ 34.1 \\ 157 \\ -51.8 \end{bmatrix}\begin{matrix} 1 \\ 2 \\ 7 \\ 8 \end{matrix} {*}10^{-4}\ [N] \qquad (19)$$

Strain and stress results induced in each finite element of the composite spring

Strains for each element ($\varepsilon_1, \varepsilon_2,$ and ε_3) are calculated using the following equations [20]:

$$\varepsilon_1=\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}=B_1\,a_1=\begin{bmatrix} 4.96 \\ 0.37 \\ -3.22 \end{bmatrix}{*}10^{-4}$$

$$\varepsilon_2=\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}=B_2\,a_2=\begin{bmatrix} 0.32 \\ 0.37 \\ 1.95 \end{bmatrix}{*}10^{-4} \qquad (20)$$

$$\varepsilon_3=\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}=B_3\,a_3=\begin{bmatrix} 4.39 \\ -10.6 \\ 8.93 \end{bmatrix}{*}10^{-4}$$

Stresses for each element σ_1, σ_2, and σ_3) are calculated using the following equations [21]:

$$\sigma_1=\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}=D_1\,\varepsilon_1=\begin{bmatrix} 39 \\ 14 \\ -8.66 \end{bmatrix}\ [MPa]$$

$$\sigma_2=\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}=D_2\,\varepsilon_2=\begin{bmatrix} 9.99 \\ 10.8 \\ 15.7 \end{bmatrix}\ [MPa] \qquad (21)$$

$$\sigma_3=\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}=D_3\,\varepsilon_3=\begin{bmatrix} 26.5 \\ -215 \\ 72.1 \end{bmatrix}\ [MPa]$$

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According to the results [Eqs. 20] the greatest deflection is about 0.59 mm and is occurred at the loaded end of the spring. The greatest normal stress [Eqs. 21] is 39 MPa for element 1 made in aluminum, and 215 MPa but negative for element 3 made in steel are found.

Conclusion

The Finite Element Method explained step by step and to inspire the understanding of the main principle. It may be accepted that the stress of 215 MPa induced in the spring is reasonable with comparing yield strength of the 9292 Silicon-Manganese of 950 MPa (Budinas and Nisbett, 2014; Spotts and Shoup, 2006). Later according to the used design factor, the dimensions of the element may be changed. It is said that the main advantage such composite beams is to be suitable for the corrosive and different environments. It is desired that the part of the spring in the corrosive atmosphere must has a very high resistance. This can be achieved for some instances by means of coating and in the other cases using the composite spring elements.
The results of the deformations show that the flexibility of the spring is too small with respect to the common springs. To obtain a high enough value, dimensions of the spring may be changed.

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